Impulsive model for the Richtmyer-Meshkov instability

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A general formula for the growth rate of the Richtmyer-Meshkov instability [R. D. Richtmyer, Commun. Pure Appl. Math. **13**, 297 (1960); E. E. Meshkov, Sov. Fluid Dyn. **4**, 101 (1969)] is derived within the framework of the impulsive model. It allows us to predict the growth rate in both heavy-light and light-heavy configurations. This formula is validated over more than 100 cases with various values of the shock strength and the adiabatic exponents. The range of validity of the impulsive model is also specified. Comparisons are performed against the results available in the literature. This expression may be reduced to the Richtmyer or the Meyer-Blewett [Phys. Fluids **15**, 753 (1972)] formulas in particular cases. Specific configurations are built in order to emphasize the differences between the latter prescriptions and the formula proposed in this paper. [S1063-651X(98)13108-2]

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I. INTRODUCTION

When two different materials are impulsively accelerated into each other by a shock wave, small perturbations of the interface first grow linearly and then evolve into nonlinear structures. This instability was theoretically discovered and described by Richtmyer [1] and experimentally confirmed by Meshkov [2]. The Richtmyer-Meshkov (RM) instability occurs in various situations, from incompressible configurations [3] to high Mach number experiments in inertial confinement fusion [4]. In the framework of the impulsive model, Richtmyer [1] proposed the following expression for the linear growth rate of the instability:

$$\frac{da}{dt}=k\Delta u A^{+}a_0^{+}, \tag{1}$$

where a is the amplitude of the perturbation, k its wave number, Δu the velocity jump across the shock wave, a_0^{+} the amplitude immediately after the shock passage, and A^{+} the Atwood number after the interaction. In the above, the Atwood number is defined as $(\rho_2-\rho_1)/(\rho_2+\rho_1)$, where ρ_1 is the density of the first shocked fluid. The preshocked and postshocked amplitudes a_0^{-} and a_0^{+}, respectively, are linked by the compression factor $1-\Delta u/W_{\text{shock}}$ [1], where W_{shock} is the speed of the incident shock wave. It is usually admitted that expression (1) gives relatively good results for light to heavy accelerations, although several exceptions to Richtmyer's formula have been found [5,6]. Richtmyer's formula (1) gives the growth rate of the instability during its linear phase using the postshocked quantities only. For heavy to light accelerations, Meyer and Blewett (MB) found, on empirical grounds, that the term a_0^{+} in Eq. (1) was to be replaced by the average of the initial unshocked and shocked amplitudes to match their numerical results [7]

$$\frac{da}{dt}=\frac{1}{2}\,k\Delta u A^{+}(a_0^{+}+a_0^{-}). \tag{2}$$

These two formulas are just prescriptions. On the other hand, Fraley [8] solved the perturbation equations for the case of a reflected shock wave. He used Laplace transforms and found a simple solution for weak shock waves. The solution for strong shock waves may be given by a power series. The results obtained from this approach were compared with Richtmyer's prescription by Mikaelian [5]. He found reasonable agreement in most cases, but he also found some configurations where Richtmyer's prescription fails. These discrepancies were attributed to compressibility effects. However, it was not possible to determine a range of validity in the parameters space.

More recently, Yang, Zhang, and Sharp [6] have presented an analysis of the Richtmyer-Meshkov instability. The linear theory is formulated and, as opposed to Fraley's work, numerically solved. Moreover, a systematic comparison with Richtmyer's prescription is carried out. They draw certain conclusions from the results obtained for the reflected shock and reflected rarefaction cases. First, the agreement between both approaches is better as the incident shock strength decreases. Second, the agreement is better as the adiabatic exponents of the fluids increase and when they are not too different. These two requirements are closely related to the compressibility effects.

An analytic theory of Richtmyer-Meshkov instability has been published by Velikovich [9] for the case of a reflected rarefaction wave. He used the same kind of techniques as those used by Fraley. Moreover, the author claims that the ''qualitative explanation of the RM instability provided by the impulsive model is therefore inadequate, regardless of the successes or failures of any prescriptions based on it.''

Wouchuk and Nishihara [10] have also recently established an analytic model for the asymptotic growth in the linear RM instability. Two different formulas are obtained whether the reflected wave is a shock or a rarefaction.

In nearly all publications, the impulsive model is still used and many configurations have been found in which it gives wrong results. The purpose of this paper is to propose a simple formulation of the impulsive model that includes both Richtmyer and MB expressions. It is first postulated and validated on more than 100 configurations taken in Refs. [5, 6, 9], which allows us to estimate the range of validity of the impulsive model. This formula is then heuristically derived by using the equation of evolution of a perturbation subjected to a Rayleigh-Taylor instability within the framework of incompressible fluids (see the Appendix). This formula reads

$$\frac{da}{dt}=\frac{1}{2}\,k\Delta u(A^{+}a_0^{+}+A^{-}a_0^{-})-\frac{1}{6}\,k\Delta u(A^{+}-A^{-})(a_0^{+}-a_0^{-}). \tag{3}$$

The second part of the right-hand side of Eq. (3) appears to be very small in most cases tested. Consequently, we shall validate the following formula [11] on numerous results available in the literature:

$$\frac{da}{dt}=\frac{1}{2}\,k\Delta u(A^{+}a_0^{+}+A^{-}a_0^{-}). \tag{4}$$

These three formulas (1), (2), and (4) may be seen in the following way. Richtmyer's prescription uses only post-shocked quantities, while Meyer and Blewett's prescription takes into account the variation of the perturbation amplitude during the interaction with the shock wave. The formula proposed in this paper takes into account both the variation of the amplitude and the Atwood number during the interaction. As a result, specific limiting cases lead to either Richtmyer's or Meyer and Blewett's prescriptions.

The outline of this paper is the following. In Sec. II formula (4) is validated on the reflected shock wave cases. In Sec. III the reflected rarefaction wave case is studied. In Sec. IV we discuss the validity of our proposal and consequently the range of applicability of the impulsive model. The derivation of the basic formula (3) is given in the Appendix.

II. CASE OF A REFLECTED SHOCK WAVE

In this section formula (4) is first validated for the case of a reflected shock wave. A comparison with the results of Fraley [8] and Mikaelian [5] (FM) is carried out. Let us first define our notations. Following Mikaelian, the normalized growth rate is defined as

$$N_{\rm GR}=\frac{\dot a}{a_0^{-}\Delta u k}. \tag{5}$$

The shock strength ϵ is defined as

$$\epsilon=1-\frac{P_0}{P_3}, \tag{6}$$

where P_0 is the initial pressure and P_3 the pressure behind the incident shock wave.

In the reflected shock wave case, Mikaelian [5] compares the growth rate obtained from Fraley's work [8] with the classical Richtmyer growth rate. This is done for several initial Atwood numbers (0.25, 0.5, 0.75, and 0.95) and various pairs of adiabatic exponents $(\gamma_1,\ \gamma_2)$: (1.1, 1.667), (1.667, 1.667), (1.1, 1.1), and (1.667, 1.1). The evolution of the normalized growth rate $N_{\rm GR}$ given either by Fraley's analysis or by the impulsive model with Richtmyer's prescription (1) is computed versus the dimensionless shock strength parameter ϵ. These graphs are plotted in Figs. 1(a)–1(d) where the dashed lines correspond to Mikaelian's results obtained from Fraley's analysis and the continuous lines to Richtmyer's prescription. In Figs. 1(e)–1(h), for the same values of parameters, the $N_{\rm GR}$ is calculated by the impulsive model with formula (4) (continuous lines) and compared with Fraley's analysis. From these comparisons, it appears clearly that, in the weak shock limit, i.e., small ϵ, all growth rates obtained from formula (4) are tangential to Mikaelian's curves. On the contrary, the normalized growth rate obtained from Richtmyer's prescription deviates from FM values even for very small shock strength parameter ϵ. In other words, the slope at the origin $dN_{\rm GR}/d\epsilon|_{\epsilon=0}$ of curves obtained from formula (4), is very close to FM's results for all cases reported in Fig. 1. The relative errors defined as $E_1=\dot a_{\rm Richtmyer}/\dot a_{\rm FM}-1$ and $E_2=\dot a_{\rm Eq.\,(4)}/\dot a_{\rm FM}-1$ for the two considered impulsive models, i.e., Richtmyer's and formula (4), are plotted versus the shock strength parameter ϵ, in Figs. 2(a) and 2(b), for the parameter values $\gamma_1=\gamma_2=1.667$ and $A^{-}=0.25$, 0.5, 0.75, and 0.95. For shock strength parameter ϵ smaller than 0.4, the relative error E_2 is less than 10% and is tangential to zero when ϵ decreases. This is not true for Richtmyer's prescription (1). These two conclusions are still valid for the other combinations of parameters. Note that such a definition of the relative error as E_1 is irrelevant as one of the growth rates goes to zero.

In Ref. [6], Yang, Zhang, and Sharp (YZS) compare the results of Richtmyer's impulsive model to those obtained from small-amplitude theory. The relative error $E_3=\dot a_{\rm Richtmyer}/\dot a_{\rm YZS}-1$ between the terminal growth rate of the linear theory and the one of the impulsive model is plotted, for various combinations of gases, versus the incident shock strength ϵ in Fig. 16 of [6] and is reproduced in Fig. 3(a). The relative error for the model presented in this paper, $E_4=\dot a_{\rm Eq.\,(4)}/\dot a_{\rm YZS}-1$, is plotted in Fig. 3(b). The conclusions previously drawn from Figs. 1 and 2 still hold in these cases. The results of the linear theory [6] and the impulsive model defined by Eq. (4) tend to each other as the incident shock strength decreases. In the weak incident shock limit, they are in better agreement than the linear theory and Richtmyer's impulsive model.

To confirm this conclusion, we have carried out a systematic comparison of the predictions of the linear theory, Richtmyer's impulsive model, and formula (4), as it was done in [6]. For these comparisons, following Yang, Zhang, and Sharp, $N_{\rm GR}$ is defined as $N_{\rm GR}=\dot a/(a_0^{-}W_{\rm shock}k)$. Varying parameters are the adiabatic exponents γ_1 and γ_2, the incident shock strength ϵ, and the preshocked density ratio $R=\rho_2/\rho_1$. The results of this comparison are presented in Table I. For each entry, the upper number is Richtmyer's impulsive model result, the second one is our impulsive model result, and the lower one is obtained from numerical simulation of YZS's linear theory. For small strength shock parameter ϵ, the normalized growth rate $N_{\rm GR}$ calculated with formula (4) is closer to the linear theory than Richtmyer's result. Furthermore, as one could expect from an incompressible model, the discrepancy between formula (4) and the linear theory increases with the shock strength parameter ϵ and the difference between the adiabatic exponents.

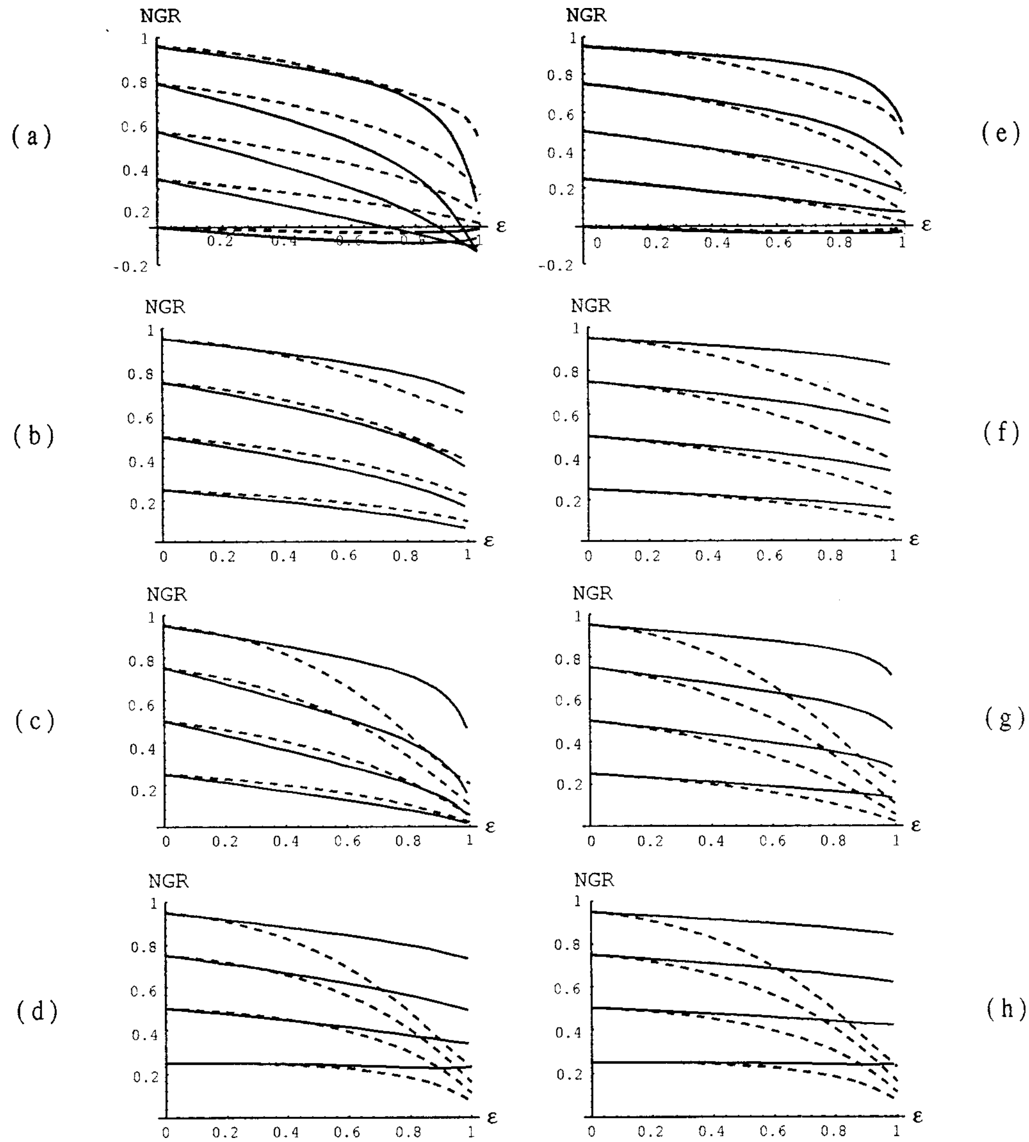

FIG. 1. (a)–(d) Normalized growth rate ($N_{\rm GR}$) according to Richtmyer's prescription (continuous lines) and according to Fraley's analysis (dashed lines). (e)–(h) $N_{\rm GR}$ according to formula (4) (continuous lines) and according to Fraley's analysis (dashed lines). For (a) and (e), the initial Atwood numbers are given by $A^-=0.0$, 0.25, 0.50, 0.75, and 0.95 and the adiabatic exponents are $\gamma_1=1.1$ and $\gamma_2=1.667$. For the other graphs, the initial Atwood numbers are given by $A^-=0.25$, 0.50, 0.75, and 0.95 and the adiabatic exponents are, for (b) and (f), $\gamma_1=1.667$ and $\gamma_2=1.667$, for (c) and (g), $\gamma_1=1.1$ and $\gamma_2=1.1$, and for (d) and (h), $\gamma_1=1.667$ and $\gamma_2=1.1$. In all cases, the curves obtained from Eq. (4) are tangential to Fraley's curves for small values of shock strength parameter ϵ, which is not true for Richtmyer's prescription.

III. CASE OF A REFLECTED RAREFACTION WAVE

The same comparisons between the linear theory formulated and numerically solved in [6] and the impulsive model can be carried out for the case of a reflected rarefaction wave. The combinations of gases are the same as those chosen in [6] and correspond to values commonly used in experiments. Let us remark that, in Ref. [6], the authors use MB's prescription (2) for the case of a reflected rarefaction wave, while they use Richtmyer's formula (1) for the reflected shock wave. We emphasize that in this paper the same formula (4) is used in both reflected rarefaction and shock wave cases. In Fig. 4(a) the relative error $E_5=\dot{a}_{\rm MB}/\dot{a}_{\rm YZS}-1$ between the terminal growth rates of the impulsive model with MB's prescription and the linear theory is plotted. The relative error E_4 for formula (4) is presented in Fig. 4(b). For the parameter values considered, the errors remain smaller than 10% for values of the shock strength parameter ϵ as large as 0.5.

As in Sec. II, we now present in Table II, for various sets of parameter values, the normalized growth rate $N_{\rm GR}$ given by impulsive models [MB's and Eq. (4)] and the linear theory (YZS's) in the case of a reflected rarefaction wave. For each entry, the upper number is MB's impulsive model result, the second one is our impulsive model result, and the lower one is obtained by numerical simulation of the linear theory (YZS's). Our formula gives here again good results in the weak shock limit for ratio γ_1/γ_2 not too different from 1.

Formula (4) can also be tested in configurations defined by Velikovich [9], who uses the failure of both Richtmyer's and MB's prescriptions to deny the relevance of the impulsive model. The parameter values are $\gamma_1=1.8$, $\gamma_2=1.45$, and $\epsilon=0.213$ and the preshocked Atwood number varies from

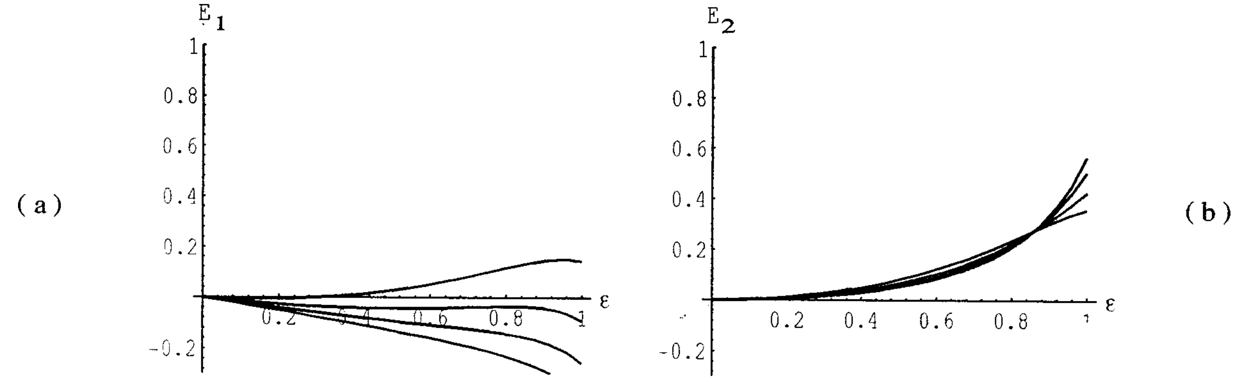

FIG. 2. (a) Relative error $E_1 = \dot{a}_{\mathrm{Richtmyer}}/\dot{a}_{\mathrm{FM}} - 1$ vs the shock strength parameter ϵ. $\dot{a}_{\mathrm{Richtmyer}}$ is the growth rate obtained from Richtmyer's prescription and $\dot{a}_{\mathrm{FM}}$ the one given by Fraley's analysis. (b) Relative error $E_2 = \dot{a}_{\mathrm{Eq.\,(4)}}/\dot{a}_{\mathrm{FM}} - 1$ vs the shock strength parameter ϵ. $\dot{a}_{\mathrm{Eq.\,(4)}}$ is the growth rate obtained from formula (4). This case corresponds to Figs. 1(b) and 1(f), i.e., the initial Atwood numbers are given by $A^- = 0.25$, 0.50, 0.75, and 0.95 and the adiabatic exponents are $\gamma_1 = 1.667$ and $\gamma_2 = 1.667$. For a given value of the shock strength parameter $\epsilon \leqslant 0.4$, the larger the initial Atwood number, the larger the relative errors E_1 and E_2. The relative error E_2 is less than 10% for ϵ smaller than 0.4. The slope at the origin of E_2 is clearly zero. This is not true for E_1.

-0.02 to 0. As shown in Fig. 5, formula (4) gives N_{GR} almost equal to the one obtained from Velikovich's analytic theory, whereas Richtmyer's and MB's prescriptions are far from the theoretical result. For example, for a preshocked Atwood number equal to 0, the relative error for N_{GR} is about 9% for our model, whereas it is about 97% for MB's prescription. Figure 5 defines a phenomenon of freeze-out that occurs when the growth rate is zero. The preshocked Atwood number for the freeze-out is found to be approximately equal to -0.0078 from Velikovich's theory and -0.0070 from formula (4). In this case, Richtmyer's and MB's prescriptions give approximately the same Atwood number, -0.0150, very far from the two previous results.

IV. DISCUSSION

In this paper we propose a formula for the growth rate of the Richtmyer-Meshkov instability in the linear phase within the framework of the impulsive model. This formula is derived in a heuristic way based on an analogy with the Rayleigh-Taylor instability (see the Appendix). We start from the ordinary differential equation, which gives the dispersion relation for a Rayleigh-Taylor instability in incompressible fluids, $\ddot{a}(t) = Agka(t)$, where g is the acceleration. This equation is applied to the Richtmyer-Meshkov instability by defining a nonzero constant acceleration during the interaction between the incident shock wave and the interface. Furthermore, the amplitude of the perturbation and the Atwood number are supposed to vary linearly during this interaction. By doing so, we obtain a formula that takes account of the variation of both the amplitude and the Atwood number. As already stated, this is not the case for the prescriptions of Richtmyer, and Meyer and Blewett. Our modeling takes into account the shock induced compression of the perturbation in a simplified way. This effect has to be distinguished from what is usually called compressibility effects. In other words, formula (4) takes into account a large part of the compression during the interaction, but takes no account of the compressibility effects after the interaction. Indeed, compressibility effects are controlled by several parameters. The most important ones seem to be the Mach number of the incident shock wave and the ratio of the adiabatic exponents. Consequently, we cannot expect formula (4) to give good results for large values of the shock strength

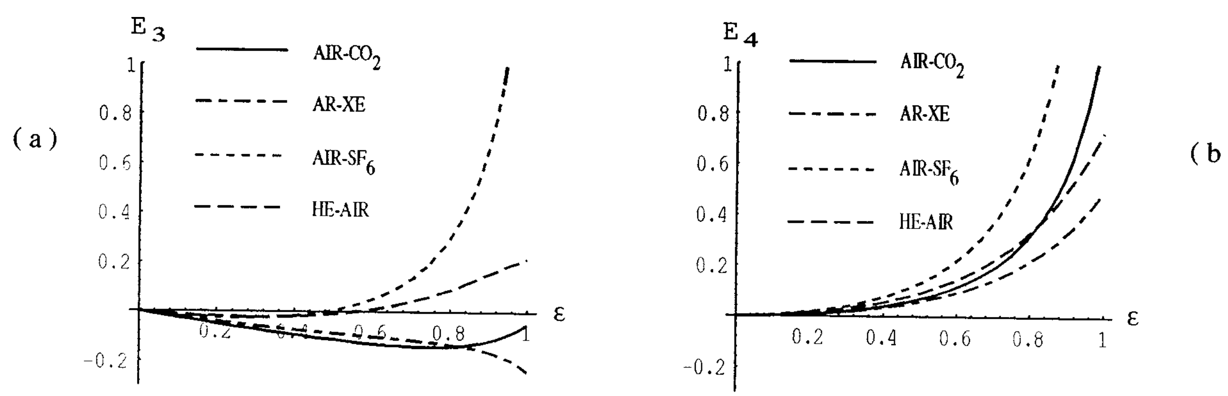

FIG. 3. (a) Relative error $E_3 = \dot{a}_{\mathrm{Richtmyer}}/\dot{a}_{\mathrm{YZS}} - 1$ between the terminal growth rate of the linear theory [4] and the one of Richtmyer's prescription vs the incident shock strength parameter ϵ for various combinations of gases. The parameters used are $\gamma_{\mathrm{He}} = \gamma_{\mathrm{Ar}} = \gamma_{\mathrm{Xe}} = 1.667$, $\gamma_{\mathrm{air}} = 1.4$, $\gamma_{CO_2} = 1.3$, $\gamma_{SF_6} = 1.0935$, $\rho_{CO_2}/\rho_{\mathrm{air}} = 1.53$, $\rho_{\mathrm{Xe}}/\rho_{\mathrm{Ar}} = 3.29$, $\rho_{SF_6}/\rho_{\mathrm{air}} = 5.1$, and $\rho_{\mathrm{air}}/\rho_{\mathrm{He}} = 7.25$. (b) Same as in (a), but for the relative error $E_4 = \dot{a}_{\mathrm{Eq.\,(4)}}/\dot{a}_{\mathrm{YZS}} - 1$. The slope at the origin of E_4 is clearly zero. This is not true for E_3. The relative error E_4 is smaller than 10% for ϵ smaller than 0.4.

TABLE I. Comparisons of normalized growth rates as given by the impulsive model with Richtmyer's prescription, the impulsive model with formula (4), and the linear theory (YZS). The first column gives the two adiabatic exponents. The second column is the shock strength parameter ϵ and the top row is the preshocked density ratio (ρ_2/ρ_1). The upper number in each entry of the table is Richtmyer's result, the second one the value obtained from formula (4), and the lower one is obtained by numerical simulation of the linear theory. The reflected wave here is a shock.

		ρ_2/ρ_1				
γ_1/γ_2	ϵ	1.1	2.0	4.0	8.0	16.0
1.1/1.1	1.0	0.001	0.016	0.050	0.095	0.14
		0.023	0.150	0.252	0.303	0.32
		0.004	0.031	0.064	0.094	0.11
	0.5	0.012	0.079	0.13	0.14	0.14
		0.017	0.104	0.16	0.17	0.15
		0.015	0.093	0.14	0.15	0.13
	0.05	0.0020	0.012	0.018	0.018	0.016
		0.0021	0.012	0.018	0.018	0.016
		0.0021	0.012	0.018	0.018	0.016
3.0/3.0	1.0	0.010	0.070	0.12	0.14	0.13
		0.017	0.103	0.16	0.17	0.15
		0.014	0.089	0.14	0.16	0.14
	0.5	0.0074	0.046	0.069	0.072	0.064
		0.0084	0.050	0.074	0.076	0.067
		0.0081	0.049	0.072	0.075	0.065
	0.05	0.00077	0.0046	0.0067	0.0068	0.0059
		0.00078	0.0046	0.0067	0.0068	0.0060
		0.00078	0.0046	0.0067	0.0068	0.0060
1.5/3.0	1.0	−0.099	−0.071	−0.012	0.054	0.10
		−0.033	0.064	0.141	0.179	0.19
		−0.0038	0.071	0.14	0.19	0.20
	0.5	−0.018	0.036	0.077	0.091	0.086
		−0.002	0.060	0.098	0.105	0.094
		0.004	0.064	0.10	0.11	0.095
	0.05	0.00099	0.0071	0.010	0.010	0.0088
		0.00114	0.0073	0.010	0.010	0.0089
		0.0012	0.0073	0.010	0.010	0.0089

(a) (b)

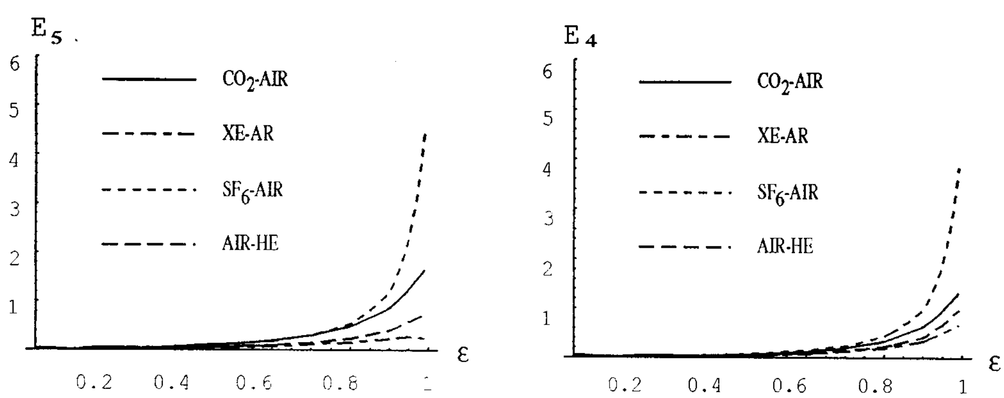

FIG. 4. (a) Relative error $E_5=\dot{a}_{\mathrm{MB}}/\dot{a}_{\mathrm{YZS}}-1$ between the terminal growth rate of the linear theory [4] and the one of the prescription of Meyer and Blewett vs the incident shock strength parameter ϵ for the same combinations of gases as in Fig. 3. (b) Same as in (a), but for the relative error $E_4=\dot{a}_{\mathrm{Eq.\,(4)}}/\dot{a}_{\mathrm{YZS}}-1$. The relative error E_4 is smaller than 10% for ϵ smaller than 0.5.

TABLE II. Same as Table I, except the reflected wave here is a rarefaction and the upper number in each entry is Meyer and Blewett's prescription.

		ρ_2/ρ_1				
γ_1/γ_2	ϵ	0.91	0.5	0.25	0.125	0.0625
1.1/1.1	1.0	−0.0085	−0.060	−0.11	−0.15	−0.18
		−0.0230	−0.172	−0.33	−0.43	−0.50
		−0.0039	−0.025	−0.042	−0.047	−0.044
	0.5	−0.017	−0.13	−0.26	−0.35	−0.41
		−0.017	−0.13	−0.26	−0.35	−0.41
		−0.016	−0.12	−0.24	−0.33	−0.39
	0.05	−0.0021	−0.017	−0.035	−0.051	−0.062
		−0.0021	−0.017	−0.035	−0.051	−0.062
		−0.0021	−0.017	−0.035	−0.051	−0.062
3.0/3.0	1.0	−0.015	−0.12	−0.23	−0.33	−0.39
		−0.017	−0.13	−0.25	−0.33	−0.42
		−0.014	−0.11	−0.22	−0.31	−0.38
	0.5	−0.0086	−0.069	−0.14	−0.20	−0.24
		−0.0086	−0.069	−0.14	−0.20	−0.24
		−0.0085	−0.068	−0.14	−0.19	−0.24
	0.05	−0.00081	−0.0065	−0.013	−0.019	−0.024
		−0.00081	−0.0065	−0.013	−0.019	−0.024
		−0.00081	−0.0066	−0.013	−0.019	−0.024
3.0/1.5	1.0	0.17	0.093	−0.029	−0.16	−0.26
		0.04	−0.086	−0.23	−0.35	−0.43
		0.016	−0.073	−0.18	−0.28	−0.36
	0.5	0.011	−0.059	−0.14	−0.20	−0.25
		−0.0001	−0.068	−0.14	−0.20	−0.25
		0.0011	−0.066	−0.14	−0.20	−0.25
	0.05	−0.00080	−0.0073	−0.015	−0.021	−0.025
		−0.00088	−0.0074	−0.015	−0.021	−0.025
		−0.00088	−0.0074	−0.015	−0.021	−0.025

parameter ϵ or for large values of the ratio of the adiabatic exponents. This quasi-incompressible assumption is not so restrictive: Experiments have been recently carried out at low shock Mach number by Jacobs, Jones, and Niederhaus [12]. From the results presented in Secs. II and III, it appears that the maximum relative errors between the various theoretical results [6,8,9] and Eq. (4) is about 10% for ϵ-parameter values smaller than 0.4 and ratios of adiabatic exponents $\gamma_{\max}/\gamma_{\min}$ smaller than 1.5, where $\gamma_{\max}$ ($\gamma_{\min}$) is the maximum (minimum) of the two adiabatic exponents. This value of 10% is an upper bound and many cases can be found with a much smaller error. The range of validity may be defined up to $\epsilon=0.5$ if a relative error of about 20% is tolerated. Once again, this is a maximum value and relative errors of 2% can often be found even for $\epsilon=0.5$ (see Tables I and II). The condition about the ratio of adiabatic exponents is not so restrictive since values of adiabatic exponents of physical gases are between 1.09 and 1.67. Furthermore, for a ratio equal to 1 but with large values of adiabatic exponents corresponding to nongaseous fluids, the relative error E_2 between formula (4) and FM's theory is still smaller than 10% for ϵ smaller than 0.4. As we can see in Fig. 6(a) for a small value of the shock strength parameter ($\epsilon=0.1$), this error tends to zero as the adiabatic exponents tend to the infinity, which corresponds to the incompressible limit. We have also plotted in Fig. 6(b) the error E_1 between Richtmyer's prescription and FM's theory. For the Atwood numbers considered, this error is at least three times larger than the error E_2. In the particular cases of Figs. 1 and 6, it turns out that for fixed values of adiabatic exponents, the difference between formula (4) and Fraley's theory reduces as the Atwood number decreases. On the other hand, for ratio of adiabatic exponents far from the value 1.5, for example, 2 as used in Tables I and II, Eq. (4) may give wrong results.

It can be noticed that the weak shock limit of the Fraley [8] and Wouchuk-Nishihara [10] analytic expressions leads to the same relation. Taking the incompressible limit for the reflected shock case with large adiabatic exponents $\gamma_1 = \gamma_2$, one obtains, at first order in ϵ,

$$\frac{\dot{a}}{a_0^- k\Delta u}=\frac{R-1}{R+1}+\frac{\epsilon}{\gamma}\frac{1-\sqrt{R}}{1+R}. \tag{7}$$

This expression is also the first-order expansion in ϵ of formula (4). So, although formula (4) is based on an analogy of the Rayleigh-Taylor instability, it provides the same expression at the incompressible limit as approximate formulas derived from exact theories.

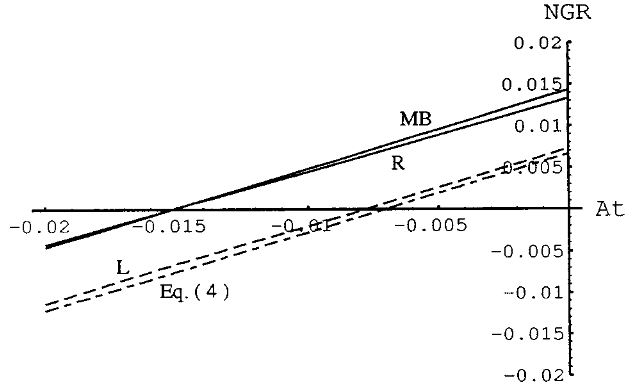

FIG. 5. Normalized growth rates given by Velikovich's linear theory (curve L), the impulsive model with the prescription of Richtmyer (curve R), the prescription of Meyer and Blewett (curve MB), and formula (4) [curve Eq. (4)] vs the initial Atwood number. The adiabatic exponents are $\gamma_1=1.8$ and $\gamma_2=1.45$. The shock strength parameter is $\epsilon=0.213$. Formula (4) gives results very close to the linear theory, which is not true for Richtmyer's and Meyer and Blewett's prescriptions.

We would like to underline that the model proposed in this paper gives a good approximation for the growth rate provided the parameters fall in the range of validity defined by $\epsilon\leqslant 0.4$ and $\gamma_{max}/\gamma_{min}\leqslant 1.5$. Indeed, no counterexample has been found and the range of validity holds in the cases of both a reflected shock wave and a reflected rarefaction wave.

On the contrary, Richtmyer's prescription is only used for reflected shock wave case, whereas MB's prescription has been introduced to handle the reflected rarefaction wave case. Moreover, nobody was able to establish a reliable range of validity for these two prescriptions. For example, in his original paper Richtmyer [1] built his prescription from configurations at $\epsilon=1$. He applied his incompressible prescription to these very compressible configurations and surprisingly obtained good results. This inconsistency has been pointed out by Yang, Zhang, and Sharp [6]: ''The agreement between the impulsive model and linear theory found by Richtmyer in the case of a strong incident shock was the accidental result of a specific choice of parameters.''

On the other hand, specific examples can show the failure of Richtmyer's prescription even for quasi-incompressible configurations in which this prescription should have worked. This point can be illustrated by the two following examples. In the case of a reflected shock wave, we consider two gases with adiabatic exponents $\gamma_1=1.667$ and $\gamma_2=1.9$ and molar masses $\mathcal{M}_1=40$ g/mol and $\mathcal{M}_2=44$ g/mol. The shock strength parameter is $\epsilon=0.4$. The values of the N_{GR} given by theory and models are presented in Table III. In this configuration, the ratio of the adiabatic exponents $\gamma_{max}/\gamma_{min}$ is 1.14, which is close to 1, and the shock strength parameter is $\epsilon=0.4$. These values should be in the range of validity of an incompressible model. Indeed, Eq. (4) produces a quite good result: The relative error with respect to Fraley's theory is less than 2%. However, Richtmyer's prescription is inaccurate: The relative error is about 40%.

TABLE III. Normalized growth rates and relative errors as given by Richtmyer's prescription, the impulsive model [Eq. (4)], and Fraley's theory. The adiabatic exponents are $\gamma_1=1.667$ and $\gamma_2=1.9$ and the molar masses are $\mathcal{M}_1=40$ g/mol and $\mathcal{M}_2=44$ g/mol. The shock strength parameter is $\epsilon=0.4$.

Source	N_{GR}	Relative error (%)
Richtmyer	0.005 14	38.3
Eq. (4)	0.008 46	1.6
Fraley	0.008 33	

The second example deals with a reflected rarefaction wave and is borrowed from Velikovich [9]. The initial Atwood number is $A^-=-0.02$, the shock strength parameter is $\epsilon=0.213$, and the adiabatic exponents are $\gamma_1=1.8$ and $\gamma_2=1.45$. The results are presented in Table IV. Here again, even in these quasi-incompressible conditions ($\epsilon=0.213$ and $\gamma_1/\gamma_2=1.24$) MB's prescription fails to give a reasonable value. The relative error is about 60%. However, Eq. (4) provides a rather good result with a 7% relative error.

In this paper a formula for the growth rate of the Richtmyer-Meshkov instability was derived within the framework of the impulsive model. It has been heuristically established from the Rayleigh-Taylor growth rate by using a nonzero constant acceleration during the interaction between the incident shock wave and the interface. All reported comparisons have shown that the revisited impulsive model pro-

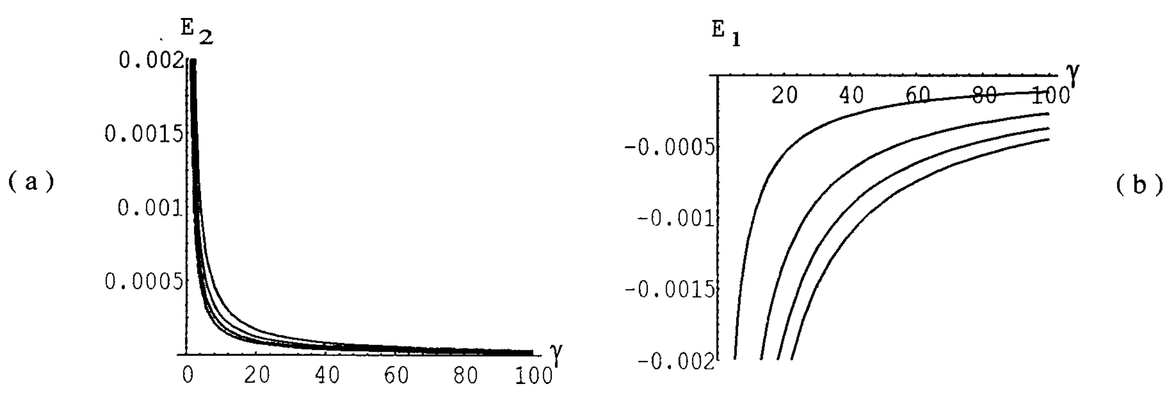

FIG. 6. (a) Relative error $E_2=\dot{a}_{\text{Eq. (4)}}/\dot{a}_{\text{FM}}-1$ between the terminal growth rate of the linear theory and the one obtained from Eq. (4) vs the adiabatic exponent γ. In this case $\gamma=\gamma_1=\gamma_2$, $\epsilon=0.1$ and the initial Atwood numbers are given by $A^-=0.25$, 0.50, 0.75, and 0.95. The error E_2 increases with the Atwood number. (b) Same as in (a), but for the relative error $E_1=\dot{a}_{\text{Richtmyer}}/\dot{a}_{\text{FM}}-1$. In that case the absolute value of the error E_1 decreases with the Atwood number. For large values of γ, i.e., in the incompressible limit, the error E_1 is at least three times larger than the error E_2.

TABLE IV. Normalized growth rates and relative errors as given by MB's prescription, the impulsive model [Eq. (4)], and Velikovich's theory. The adiabatic exponents are $\gamma_1=1.80$ and $\gamma_2=1.45$ and the initial Atwood number is $A^-=-0.02$. The shock strength parameter is $\epsilon=0.213$.

Source	N_{GR}	Relative error (%)
MB	-0.0045	60
Eq. (4)	-0.0121	7
Velikovich	-0.0113	

duces a good estimate of the growth rate of the Richtmyer-Meshkov instability provided it is used within its range of validity. Nowadays, approximate formulas derived from exact theories [8–10] are available. However, it was useful to understand and explain the failures of the impulsive model noticed in the literature for years [5,6,9]. Indeed, the revisited impulsive model gives good results in both heavy-light and light-heavy configurations provided that it is used within its range of validity, i.e., $\epsilon\leqslant 0.4$ and $\gamma_{max}/\gamma_{min}\leqslant 1.5$, which corresponds to nearly incompressible flows.

APPENDIX: DERIVATION OF THE FORMULA FOR THE GROWTH RATE OF THE RM INSTABILITY

During the interaction between a monomode perturbation and a shock wave, the interface is accelerated and compressed. The acceleration begins when the shock wave hits the interface. It finishes when the incident shock wave has gone right through the perturbation. If the initial peak to peak amplitude of the perturbation is a_0^-, the acceleration occurs between $t=-a_0^-/2W_{shock}$ and $a_0^-/2W_{shock}$, where W_{shock} is the speed of the incident shock wave. The instant $t=0$ corresponds to the moment the shock strikes the middle of the perturbation. Furthermore, we do not model the shock wave as a pure step function but as a regularized function that extends over a small thickness h. This thickness h is taken to be much smaller than the amplitude a_0^-. The effect of the acceleration is to transmit the fluids a velocity Δu that we consider constant. The acceleration g is modeled as a nonzero constant function during the action of the incident shock wave

$$g(t)=\frac{W_{shock}\Delta u}{a_0^-}\,Y_{-a_0^-/2W_{shock}}(t)[1-Y_{+a_0^-/2W_{shock}}(t)], \tag{A1}$$

where $Y_{t_0}(t)$ is a regularized Heaviside function centered at $t=t_0$.

During the interaction between the incident shock wave and the perturbation the amplitude varies from a_0^- to a_0^+. As the velocity of the interface is taken to be constant, the evolution of the amplitude $a(t)$ versus time is linear during the interaction. Its overall evolution can be modeled by

$$a(t)=(1-Y_-)a_0^-+Y_-(1-Y_+)y(t)+Y_+[a_0^++f(t)], \tag{A2}$$

where

$$y(t)=\frac{W_{shock}}{a_0^-}\,(a_0^+-a_0^-)t+\frac{1}{2}\,(a_0^++a_0^-).$$

In this expression, $Y_\pm=Y_{\pm a_0^-/2W_{shock}}(t)$ and $f(t)$ is a continuous function taken to be constant and equal to $f(0)$ for $t\leqslant 0$ and to $a(t)$ for $t\gg a_0^-/2W_{shock}$.

During the interaction between the incident shock wave and the perturbation, the Atwood number varies from A^- to A^+. However, the evolution of the Atwood number is quite complicated. We shall approximate it by a linear function

$$A(t)=(1-Y_-)A^-+Y_-(1-Y_+)z(t)+Y_+A^+ \tag{A3}$$

where

$$z(t)=\frac{W_{shock}}{a_0^-}\,(A^+-A^-)t+\frac{1}{2}\,(A^++A^-).$$

We start from the ordinary differential equation, which gives the dispersion relation for a Rayleigh-Taylor instability in incompressible fluids, $\ddot{a}(t)=Agka(t)$, where k is the wave number of the perturbation. By introducing the expressions for the acceleration $g(t)$, the Atwood number $A(t)$, and the amplitude $a(t)$, the previous differential equation can be integrated from $t=-\infty$ to $t\gg a_0^-/2W_{shock}$,

$$\int_{-\infty}^{t}g(t')A(t')ka(t')dt'=\int_{-\infty}^{t}\ddot{a}(t')dt'. \tag{A4}$$

Equation (A4) gives, for $t\gg a_0^-/2W_{shock}$,

$$\dot{a}(t)\approx\tfrac{1}{2}k\Delta u[A^-(\tfrac{2}{3}a_0^-+\tfrac{1}{3}a_0^+)+A^+(\tfrac{2}{3}a_0^++\tfrac{1}{3}a_0^-)]. \tag{A5}$$

Finally, we propose the following formula for the linear growth rate of the Richtmyer-Meshkov instability:

$$\frac{da}{dt}=\frac{1}{2}\,k\Delta u(A^+a_0^++A^-a_0^-)-\frac{1}{6}\,k\Delta u(A^+-A^-)(a_0^+-a_0^-). \tag{A6}$$

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